AE 3613

Orbits 1

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**HW #02**

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**Problem 1**

Looking at the Europa Clipper mission scheduled to launch around October 10th this year (2024) and arrive at Jupiter’s moon, Europa, and land sometime in 2030, the projected path can be a bit intimidating and difficult to analyze. To simply basic calculations, it can be assumed that rather than the actual projected flight path (figure 1), the Clipper will take a much simpler path (figure 2). For ease of calculation, it can also be assumed that the Clipper can travel directly through celestial bodies, as well as that the only bodies exerting a significant gravitational force on the Clipper are the Sun, Earth, Jupiter, and Europa.

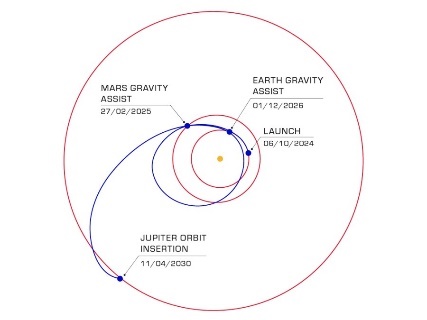


Figure 1

A diagram of a diagram

Description automatically generated with medium confidence

Figure 2

By defining sphere of influence (SOI) as when the distance at which one body is exerting a larger gravitational force than the other, the method of determining where the SOI ends is to use Newton’s Universal Law of Gravitation, , for both bodies being analyzed. Since the SOI of one body ends where the other begins, at the exact point where both SOI’s end the gravitational forces will be equal. So, a hypothetical system would look like , , and , so and divide out leaving only , where and are the SOI of bodies 1 and 2 respectively. If one of the distances is put into terms of the other, say , where r is a known value, the equation can be rearranged into a quadratic equation, **(1)**. Using quadratic formula with , , and , can be solved for.

For the SOI’s being found here, I. Earth vs the Sun, II. the Sun vs Jupiter, and III. Jupiter vs Europa, the values for mass and distance can be plugged in and the SOI’s can be solved for.

* First, define necessary information:

\*\*NOTE\*\* Subscript E denotes Europa and subscript i denotes the Europa Clipper vessel)

* + *Masses (in kilograms)*
    - , , ,
  + *Distances (in Astronomical Units)*
    - , ,
  + *Relative distances (AU)*
    - , , ,

1. Plugging in values for , , , and into **(1)**, the equation becomes , where , , and . Using MATLAB to solve the quadratic, it is found that AU, and AU, or that km and km.
2. Plugging in values for , , , and into **(1)**, the equation becomes , where , , and . Using MATLAB to solve the quadratic, it is found that AU and AU or that km and km.
3. Plugging in values for , , , and into **(1)**, the equation becomes , where , , and . Using MATLAB to solve the quadratic, it is found that AU and AU or that km and km.

**Problem 2**

The “Tilt-a-Tot” (figure 3) is a new and possibly dangerous toy being introduced on playgrounds near you. It’s essentially a teeter-totter expanded to three dimensions, and the prototype consists of a 5-meter by 5-meter surface that has been placed on a fulcrum located at the geometric center of the surface.A diagram of a square with lines and points

Description automatically generated

Figure 3

4 toddlers, toddler A, toddler B, toddler C, and toddler D are testing this prototype and weigh a respective 20 lbf, 25 lbf, 22 lbf, and 40 lbf. The toddlers have already been modeled on the “Tilt-a-Tot” in figure 3. To find the center of mass (C.M) of the loaded “Tilt-a-Tot”, the equation **,** where is the vector location of the C.M relative to the origin (which has been placed at the geometric center of the “Tilt-a-Tot”), is the total mass of the system, and is the summation of the individual masses multiplied by their position relative to the origin can be used. First it is rearranged to  **(2)** to solve for the vector location of the C.M.

* Define the position vectors of each toddler relative to the origin (in meters)
  + - , **, ,**
* Define the toddler’s weights (in lbf)
  + - , , , ,
* \*\*NOTE\*\* it is not necessary to find the toddler’s mass, as the gravitational will divide out when the values are plugged into **(2)**.
* Break **(2)** down into **x** and **y** (**i** and **j**) components
  + By plugging in the known values, the **x** component is found to be meters, and the **y** component is found to be meters, or .
* Due to how the coordinate axes were defined, the distance from the geometric center is simply the magnitude of vector .
  + By plugging the values for and into the equation for magnitude, it is found that meters.

If toddler B is to start moving towards toddler C at a rate of 20 centimeters per second (0.20 meters per second), the center of mass can and its distance from the geometric center can now be plotted as a function of time. To do this, the position vector must be rewritten as a function of time using toddler B’s velocity relative to toddler C.

* Rewrite vector as a function oftime,
  + First find vector
    - , or
  + Now find vector as a function of time, using basic kinematics equations
    - * ,
      * Solving for t, it is found that it will take toddler B 25 seconds to reach toddler C.
  + Using , find
    - **,** or
* Plug the new into the and and find the magnitude of the new C.M as a function of time
  + - * \*\*NOTE\*\* since the **y** component of is zero, the **y** component of does NOT change with time.
* Finally plot the results using MATLAB
  + - ~ figure 4
    - ~ figure 5

A graph with a line

Description automatically generated

Figure 4

A graph with a red line

Description automatically generated

Figure 5

Toddler B probably didn’t think this decision through, as by crawling over to the other side of the “Tilt-a-Tot”, they will have made it much harder to do anything on the toy. This is due to the center of mass having shifted so far in the negative X direction, and this will make it so that Toddler A is stuck in the air and can’t play.

**Problem 3**

TCM’s, or Trajectory Correction Maneuver, are propulsive maneuvers spacecrafts due to make a small adjustment to its flight path. These maneuvers are typically done with cold gas thrusters, essentially just releasing compressed gases from a nozzle. These thrusters typically use gases like nitrogen, hydrogen, helium, or other lightweight gases, and the lack of adding heat or chemical reactions makes these thrusters less powerful, and much easier control; perfect for TCM’s. When a TCM goes wrong different things can happen depending on what the TCM was being used for. If it were being used for correcting a small error in the middle of its path somewhere, it would need to be corrected using more of the gas from the cold gas thrusters. If it were being used to maintain orbit, the craft could spiral out of orbit and crash onto the body it was orbiting. If it were being used to dock at a space station, it could result in failure to dock as well as damaging the spacecraft. If the thrusters fail, it could also prevent the craft from making TCM’s in the future, which could completely prevent the craft from following its desired path. While it’s possible that a TCM fail is part of why the Starliner vessel is “stuck” at the ISS, it’s unlikely that it’s the root cause. Technically it would be because five of the cold gas thrusters used for these TCM’s overheated and stopped functioning rather than a failed maneuver. Due to the overheating of these thrusters (which I find to be rather ironic due to the nature of cold gas thrusters and Gay-Lussac's Law), the seals failed, and the helium gas being used began to leak. This ultimately makes any travel with the craft very risky as it would need to function with less attitude adjustors than it was designed to function with.

**Citations**

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**MATLAB**

% Define Weights (Weights can be used rather than masses because

% gravitational constant cancels out in center mass equation

W\_a = 20;

W\_b = 25;

W\_c = 22;

W\_d = 40;

W = W\_a + W\_b + W\_c + W\_d;

% Define Vectors x and y components (relative to geometric center, o)

r\_ax = 2.5;

r\_ay = 2.5;

r\_bx = 2.5;

r\_by = -2.5;

r\_cx = -2.5;

r\_cy = -2.5;

r\_dx = -2.5;

r\_dy = 2.5;

% Time range from 0 to 25 seconds

t = linspace(0, 25, 1000); % 1000 points for smooth plots

% Replace these with the actual functions later

RCM\_x = (((W\_a \* r\_ax) + (W\_c \* r\_cx) + (W\_d \* r\_dx)) + (W\_b \* (2.5 - (0.2 \* t)))) ./ W;

RCM\_y = (((W\_a \* r\_ay) + (W\_b \* r\_by) + (W\_c \* r\_cy) + (W\_d \* r\_dy))) ./ W .\* ones(size(t));

distance\_to\_center = sqrt(RCM\_x.^2 + RCM\_y.^2);

figure;

% Plot 1: X and Y Components of Center Mass vs Time

subplot(2, 1, 1); % Creates a 2x1 grid of plots, and this is the first plot

plot(t, RCM\_x, 'b', 'LineWidth', 1.5); % Plot X component in blue

hold on; % Keep the current plot to overlay the Y component

plot(t, RCM\_y, 'g--', 'LineWidth', 1.5); % Plot Y component in green dashed line

title('Center Mass (X and Y components) vs Time');

xlabel('Time (s)');

ylabel('Center Mass');

legend('X component', 'Y component');

grid on; % Add grid to the plot

hold off; % Release the plot hold

% Plot 2: Distance From Center Mass to Geometric Center vs Time

subplot(2, 1, 2); % This is the second plot in the 2x1 grid

plot(t, distance\_to\_center, 'r', 'LineWidth', 1.5); % Plot in red with line width of 1.5

title('Distance From Center Mass to Geometric Center vs Time');

xlabel('Time (s)');

ylabel('Distance to Geometric Center');

grid on; % Add grid to the plot